CHAPTER 8

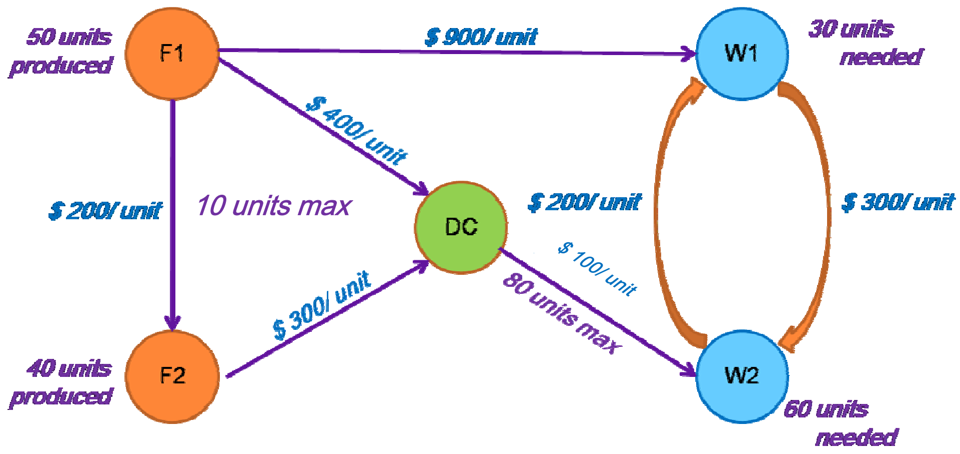
TRANSPORTATION PROBLEMS
ปัญหาการขนส่ง

TRANSPORTATION PROBLEMS (1)

- Transportation problems are for optimizing the cost of transportation by using Linear Programming (LP).
- However, transportation problem has a special characteristic.
- Therefore, it can be solved easily by using its unique method.

Transportation Problems (2)

<u>ปัญหาการขนส่ง</u> เป็นปัญหาของ Linear Programming รูปแบบหนึ่ง ซึ่งมีความพิเศษกว่าปัญหาที่ได้เรียนมาก่อนหน้านี้ ปัญหานี้ต้องการหาค่าใช้จ่ายในการขนส่งที่ต่ำที่สุด หรือหาเวลาในการขนส่งน้อยที่สุด จาก Source ไปยัง Destination ต่าง ๆ

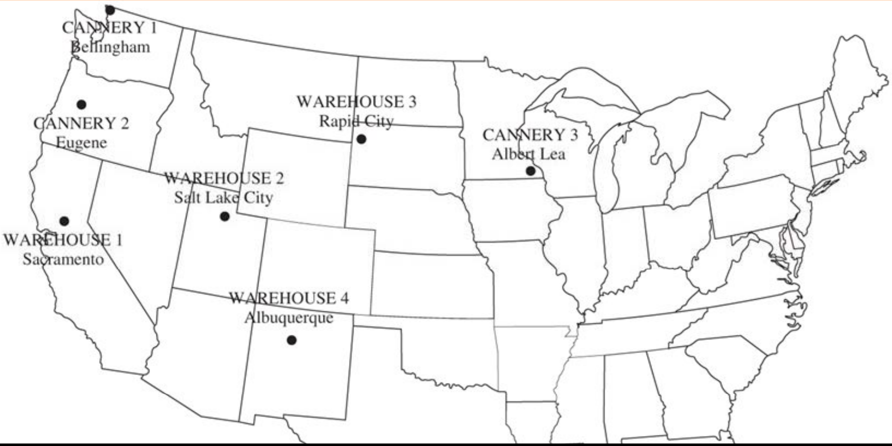


Transportation Problems (3)

The Transportation Problem Model

- เป็นการแก้ปัญหาการขนส่ง โดยการกระจายสินค้าจากแหล่งผลิต(*sources)* ไปยังลูกค้า (*destinations*) โดยมีเป้าหมายเพื่อที่จะทำให้ต้นทุนการกระจายสินค้าโดยรวม ต่ำที่สุด (to <u>*minimize the total distribution cost)*</u>
- Each source has a certain *supply* of units to distribute to the destinations, and each destination has a certain *demand* for units to be received from the sources.

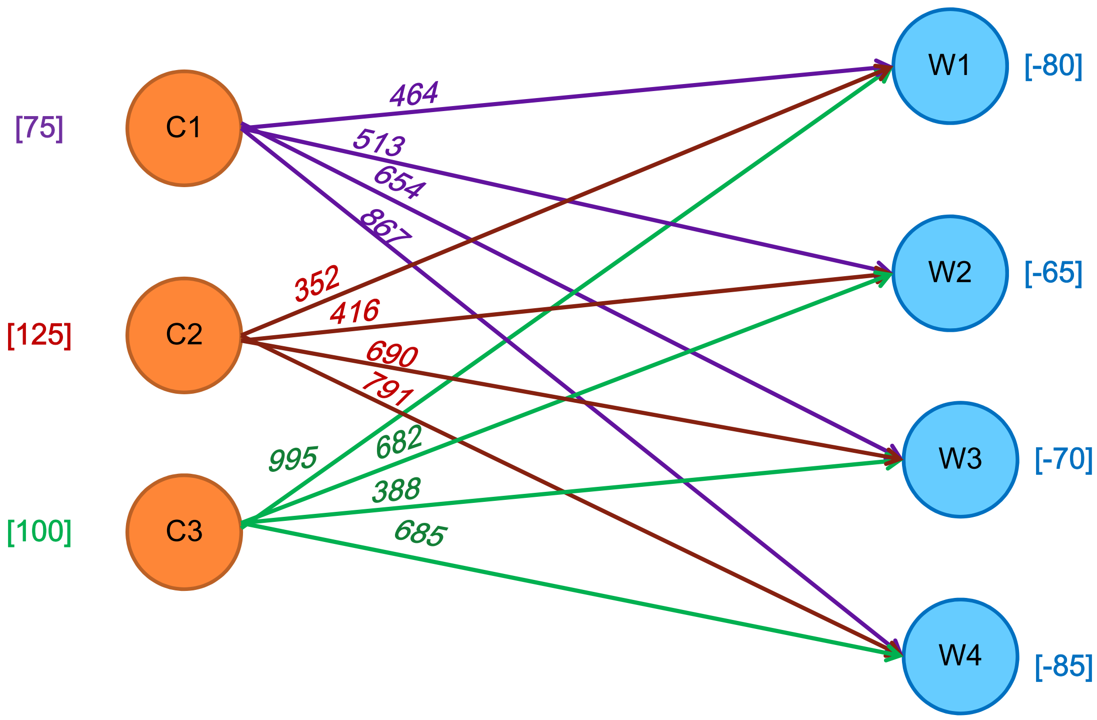
Transportation Problems (4)



	Shipping cost ($) per Truckload				Output
	Warehouse				
Cannery	1	2	3	4	
1	464	513	654	867	75
2	352	416	690	791	125
3	995	682	388	685	100
Allocation	80	65	70	85	

Shipping Data for P&T Co.

Transportation Problems (5)



Constraint coefficient of P&T Co. (6)

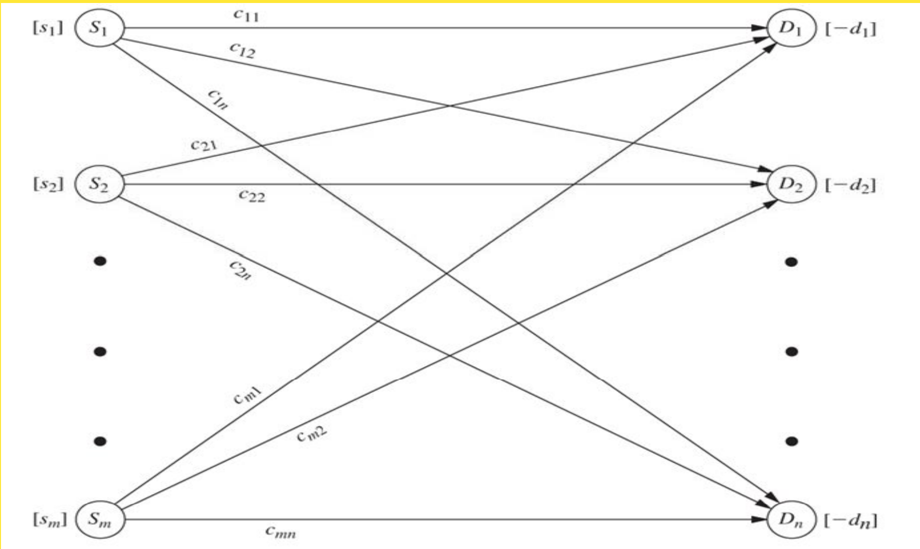
TABLE 8.3 Constraint coefficients for P & T Co.

Coefficient of:

	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	
A =	1	1	1	1									Cannery constraints
					1	1	1	1					
									1	1	1	1	
	1				1				1				Warehouse constraints
		1				1				1			
			1				1				1		
				1				1				1	

Prototype Example	General problem
จำนวนTruckloads ของถั่วกระป๋อง	จำนวนหน่วยของสินค้า
โรงงานผลิตถั่วกระป๋อง 3 แห่ง	แหล่งผลิต *m* แห่ง
คลังสินค้าถั่วกระป๋อง 4 แห่ง	คลังสินค้า *n* แห่ง
Output from canneries *i*	จำนวนสินค้าที่ผลิตได้จาก Source i, S_i
Allocation to warehouse *j*	ความต้องการที่ destination j, D_j
Shipping cost per truckload from canneries *i* to warehouse *j*	ต้นทุนในการขนส่ง C_{ij} จากsource i ไป destination j

Network of Transportation Problem (7)



$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} Cij\ Xij$$

$$\text{Subject to} \sum_{j=1}^{n} X_{ij} = S_i \quad for\ i = 1,2,3,\ldots.m,$$

$$\text{Subject to} \sum_{i=1}^{m} X_{ij} = d_j \quad for\ j = 1,2,3,\ldots.n$$

$$X_{ij} \geq 0 \ for\ all\ i\ and\ j$$

A sample with a dummy destination and source (8)

โดยเงื่อนไขปกติแล้ว จะกำหนดให้ ความสามารถในการผลิตของทุก ๆ แหล่ง เท่ากับความต้องการของสินค้าของทุก ๆ คลังสินค้า

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$$

สำหรับปัญหาขนส่งทั่วไป ที่ $\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} d_j$

จะทำให้เท่ากันโดยการเพิ่มแหล่งผลิตหลอก ๆ หรือเพิ่มคลังสินค้าหลอก (Dummy destination or Dummy source)

$$\sum_{j=1}^{n} d_j - \sum_{i=1}^{m} S_i > 0$$ จะเพิ่มแหล่งผลิต

$$\sum_{i=1}^{m} S_i - \sum_{j=1}^{n} d_j > 0$$ จะเพิ่มคลังสินค้า

Transportation Problems (9)

- How to solve transportation problem?
- There are two steps.
- The first step: Finding the initial solution
  - Northwest Corner Method
  - Least Cost Method
  - Vogel's Approximation Method
- The second step: Optimality Test
  - If the initial solution is not optimal, the solution is improved until the optimality test is valid.

Transportation Problems (10)

- How to set up a transportation simplex tableau?

TABLE 8.5 Parameter table for the transportation problem

		Cost per Unit Distributed				
		Destination				
		1	2	⋯	n	Supply
Source	1	c_{11}	c_{12}	⋯	c_{1n}	s_1
	2	c_{21}	c_{22}	⋯	c_{2n}	s_2
	⋮					⋮
	m	c_{m1}	c_{m2}	⋯	c_{mn}	s_m
Demand		d_1	d_2	⋯	d_n	

Parameter table for transportation problem

Transportation Problems (11)

- Example 1: A firm has 3 factories; A, B and C, shipping product to destinations; 1, 2, 3 and 4. The detail are summarized on the table.

Factory	Supply (units)
A	500
B	700
C	800

Destination	Demand (units)
1	400
2	900
3	200
4	500

TRANSPORTATION PROBLEMS (12)

- Transportation Cost:

From	To			
	1	2	3	4
A	12	13	4	6
B	6	4	10	11
C	10	9	12	4

- Check: Total Demand = Total Supply

  400+900+200+500 = 500+700+800

  2000 = 2000 ➔ OK

TRANSPORTATION PROBLEMS (13)

Set up transportation tableau:

Source	Destination				Supply
	1	2	3	4	
A	12	13	4	6	500
B	6	4	10	11	700
C	10	9	12	4	800
Demand	400	900	200	500	

- No. of basic variable = m+n-1 = 3+4-1 = 6

1. Northwest Corner Rule (14)

TABLE 8.16 Initial BF solution from the Northwest Corner Rule

		Destination 1	2	3	4	5	Supply	u_i
Source	1	16 (30)	16 (20)	13	22	17	50	
	2	14	14 (0)	13 (60)	19	15	60	
	3	19	19	20 (10)	23 (30)	*M* (10)	50	
	4(*D*)	*M*	0	*M*	0	0 (50)	50	
Demand		30	20	70	30	60	$Z = 2{,}470 + 10M$	
	v_j							

- ไม่คำนึงถึงcost เป็นวิธีที่ง่ายที่สุด
- เริ่มต้นจากมุมบน ซ้ายสุดก่อน ใส่ค่า x_{ij} ลงไปให้มากที่สุดก่อน แล้วจึงไล่ลงมาเรื่อย ๆ
- พิจารณาถึง source และ destination ที่เหลืออยู่
- จำนวนตัวแปรที่เป็น basic variable ต้องมีจำนวน = m+n-1

Transportation Problems (15)

- 1st Step: Finding initial solution
  - By Northwest Corner Method:

BV = 6 → OK

Source	Destination 1	2	3	4	Supply
A	12	13	4	6	500
B	6	4	10	11	700
C	10	9	12	4	800
Demand	400	900	200	500	

Solution:
$Z = 14200$
BV:
$X_{A1} = 400$
$X_{A2} = 100$
$X_{B2} = 700$
$X_{C2} = 100$
$X_{C3} = 200$
$X_{C4} = 500$
Non-BV:
Other $X_{ij} = 0$

TRANSPORTATION PROBLEMS (16)

- 1st Step: Finding initial solution
  - By Least Cost Method:

BV = 6 ➔ OK

Source	Destination				Supply
	1	2	3	4	
A					500
B					700
C					800
Demand	400	900	200	500	

Solution:
Z = 12000
BV:
$X_{A1} = 300$
$X_{A3} = 200$
$X_{B2} = 700$
$X_{C1} = 100$
$X_{C2} = 200$
$X_{C4} = 500$
Non-BV:
Other $X_{ij} = 0$

TRANSPORTATION PROBLEMS (17)

- By Vogel's Approximation Method

1. หาค่าความแตกต่างระหว่าง cost ของแต่ละแถว และคอลัมน์
2. เลือกแถวหรือคอลัมน์ที่มีค่าความแตกต่างมากที่สุด และทำการจัดการขนส่งให้มากที่สุด ให้แก่ cell ที่มีต้นทุนต่ำที่สุด ในแถวหรือคอลัมน์นั้น ๆ Select row/ column with the largest penalty then select the cell that contain the lowest cost and assign the largest possible value of X_{ij}.
3. Eliminate that row or column and recalculate the penalty and do the same process until no. of BV. meet the condition.

Transportation Problems (18)

- By Vogel's Approximation Method

Source	Destination 1	2	3	4	Supply	1	2	3	4	5
A	12	13	4 200	6 300	500	2	6			
B	6	4 700	10	11	700	2	2	2	2	
C	10 400	9 200	12	4 200	800	5	5	1	1	
Demand	400	200	200	500						
1	4	5	6	2						
2	4	5		2						
3	4	5		7						
4	2	5								
5										

No. of BV = 6
→ OK

Transportation Problems (19)

Source	Destination 1	2	3	4	Supply
A	12	13	4 200	6 300	500
B	6	4 700	10	11	700
C	10 400	9 200	12	4 200	800
Demand	400	900	200	500	

- Solution: Z = 12000

  BV: $X_{A3} = 200$, $X_{A4} = 300$, $X_{B2} = 700$, $X_{C1} = 400$, $X_{C2} = 200$, $X_{C4} = 200$

  Non-BV: Other $X_{ij} = 0$

Transportation Problems (20)

- **2nd Step: Optimality Test**
  - A basic feasible solution is optimal if and only if

    $C_{ij} - u_i - v_j \geq 0$

    for every (i, j) such that X_{ij} is <u>nonbasic</u>.

    And, $C_{ij} = u_i + v_j$

    for each (i, j) such that X_{ij} is basic.

  - How to find u_i and v_j for each i and j?
    - From BV., Assign $u_i = 0$ when that row has the maximum no. of allocations.

Transportation Problems (20)

- 2nd Step: Optimality Test
  - Start from VAM's solution

Source	Destination 1	2	3	4	Supply
A	12	13	4 / **200**	6 / **300**	500
B	6	4 / **700**	10	11	700
C	10 / **400**	9 / **200**	12	4 / **200**	800
Demand	400	900	200	500	

BV:

X_{A3} : $4 = u_A + v_3$; set $u_C = 0$

X_{A4} : $6 = u_A + v_4$ $v_1 = 10$,

X_{B2} : $4 = u_B + v_2$ $v_2 = 9$,

X_{C1} : $10 = u_C + v_1$ $v_4 = 4$,

X_{C2} : $9 = u_C + v_2$ $u_A = 2$,

X_{C4} : $4 = u_C + v_4$ $u_B = -5$,

$v_3 = 2$

NBV: ***$C_{ij} - u_i - v_j \geq 0$ ***

X_{A1} : $12 - u_A - v_1 = 12 - 2 - 10 =$ **0**

X_{A2} : $13 - u_A - v_2 = 13 - 2 - 9 = 2$ **OK**

X_{B1} : $6 - u_B - v_1 = 6 - (-5) - 10 = 1$

X_{B3} : $10 - u_B - v_3 = 10 - (-5) - 2 = 13$

X_{B4} : $11 - u_B - v_4 = 11 - (-5) - 4 = 12$

X_{C3} : $12 - u_c - v_3 = 12 - 0 - 2 = 10$

The solution from VAM is optimal.

TRANSPORTATION PROBLEMS (21)

- 2nd Step: Optimality Test
  - How about when the initial solution is not optimal?
  - Basic feasible solution from NW:

Source	Destination 1	2	3	4	Supply
A	12 / 400	13 / 100	4	6	500
B	6	4 / 700	10	11	700
C	10	9 / 100	12 / 200	4 / 500	800
Demand	400	900	200	500	

TRANSPORTATION PROBLEMS (22)

Source	Destination 1	2	3	4	Supply
A	12 / 400	13 / 100	4	6	500
B	6	4 / 700	10	11	700
C	10	9 / 100	12 / 200	4 / 500	800
Demand	400	900	200	500	

BV:

X_{A1} : $12 = u_A + v_1$; set $u_C = 0$
X_{A2} : $13 = u_A + v_2$ $v_2 = 9$,
X_{B2} : $4 = u_B + v_2$ $v_3 = 12$,
X_{C2} : $9 = u_C + v_2$ $v_4 = 4$,
X_{C3} : $12 = u_C + v_3$ $u_A = 3$,
X_{C4} : $4 = u_C + v_4$ $u_B = -5$,
$v_1 = 9$

NBV: ***$C_{ij} - u_i - v_j \geq 0$ ***

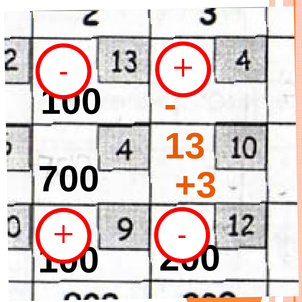
X_{A3} : $4 - u_A - v_3 = 4 - 3 - 12 = -13$
X_{A4} : $6 - u_A - v_4 = 6 - 3 - 4 = -1$
X_{B1} : $6 - u_B - v_1 = 6 - (-5) - 9 = 2$
X_{B3} : $10 - u_B - v_3 = 10 - (-5) - 12 = 3$
X_{B4} : $11 - u_B - v_4 = 11 - (-5) - 4 = 12$
X_{C1} : $10 - u_c - v_1 = 12 - 0 - 9 = 3$

This solution has to be improved!!

TRANSPORTATION PROBLEMS (23)

Source	Destination 1	2	3	4	Supply
A	[12] 400	[13] 100	[4] -13	[6] - 1	500
B	[6] +2	[4] 700	[10] +3	[11] +12	700
C	[10] +3	[9] 100	[12] 200	[4] 500	800
Demand	400	900	200	500	

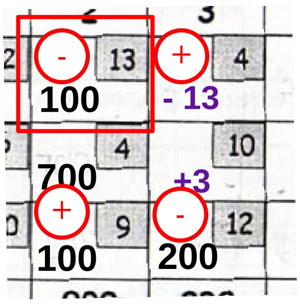
1) Select the index that has largest minus value to be entering variable. (This implies to how much that Z can be improved for Min Prob.)



2) Use the cell from 1), set closed loop from that cell by having BV on all corner points.

3) Set the starting point as + sign and the next corner as – sign until complete all corners.

TRANSPORTATION PROBLEMS (24)



Source	Destination 1	2	3	4	Supply
A	[12] 400	[13] 100	[4] 100	[6]	500
B	[6]	[4] 700	[10]	[11]	700
C	[10]	[9] 200 (was 100)	[12] 100 (was 200)	[4] 500	800
Demand	400	900	200	500	

4) See the minus corner, select the minimum value of BV.

5) Use the value from 4) to add and reduce the BVs following the sign.

6) Recalculate indexes of each non-BV again and check the optimality test until the optimality condition is valid.

TRANSPORTATION PROBLEMS (25)

- Unbalanced Case:
  - Case 1: Demand > Supply
  - Dummy Source: Dummy Supply = $\sum_{j=1}^{n} d_j - \sum_{i=1}^{m} S_i$

Sum of Supplies ≠ Sum of Demands

Source	Destination				Supply
	1	2	3	4	
A	12	13	4	6	500
B	6	4	10	11	700
Demand	400	900	200	500	

2000

Dummy Source

Source	Destination				Supply
	1	2	3	4	
A	12	13	4	6	500
B	6	4	10	11	700
Dummy	0	0	0	0	800
Demand	400	900	200	500	2000

TRANSPORTATION PROBLEMS (26)

- Unbalanced Case:
  - Case 2: Supply > Demand
  - Dummy Destination: Dummy Demand = $\sum_{i=1}^{m} S_i - \sum_{j=1}^{n} d_j$

Sum of Supplies ≠ Sum of Demands

Source	Destination			Supply
	1	2	3	
A	12	13	4	500
B	6	4	10	700
C	10	9	4	800
Demand	400	900	200	

1500

Dummy Destination

Source	Destination				Supply
	1	2	3	Dummy	
A	12	13	4	0	500
B	6	4	10	0	700
C	10	9	4	0	800
Demand	400	900	200	500	2000

AN EXAMPLE WITH A DUMMY DESTINATION (27)

TABLE 8.7 Production scheduling data for Northern Airplane Co.

Month	Scheduled Installations	Maximum Production	Unit Cost* of Production	Unit Cost* of Storage
1	10	25	1.08	0.015
2	15	35	1.11	0.015
3	25	30	1.10	0.015
4	20	10	1.13	

*Cost is expressed in millions of dollars.

TABLE 8.8 Incomplete parameter table for Northern Airplane Co.

		Cost per Unit Distributed				
		Destination				
		1	2	3	4	Supply
Source	1	1.080	1.095	1.110	1.125	?
	2	?	1.110	1.125	1.140	?
	3	?	?	1.100	1.115	?
	4	?	?	?	1.130	?
Demand		10	15	25	20	

AN EXAMPLE WITH A DUMMY DESTINATION (28)

TABLE 8.9 Complete parameter table for Northern Airplane Co.

		Cost per Unit Distributed					
		Destination					
		1	2	3	4	5(*D*)	Supply
Source	1	1.080	1.095	1.110	1.125	0	25
	2	*M*	1.110	1.125	1.140	0	35
	3	*M*	*M*	1.100	1.115	0	30
	4	*M*	*M*	*M*	1.130	0	10
Demand		10	15	25	20	30	

An example with a dummy source (29)

■ **TABLE 8.10** Water resources data for Metro Water District

	Cost (Tens of Dollars) per Acre Foot				
	Berdoo	**Los Devils**	**San Go**	**Hollyglass**	**Supply**
Colombo River	16	13	22	17	50
Sacron River	14	13	19	15	60
Calorie River	19	20	23	—	50
Minimum needed	30	70	0	10	(in units of 1 million acre feet)
Requested	50	70	30	∞	

■ **TABLE 8.11** Parameter table without minimum needs for Metro Water District

		Cost (Tens of Millions of Dollars) per Unit Distributed				
		Destination				
		Berdoo	**Los Devils**	**San Go**	**Hollyglass**	**Supply**
Source	Colombo River	16	13	22	17	50
	Sacron River	14	13	19	15	60
	Calorie River	19	20	23	*M*	50
	Dummy	0	0	0	0	50
Demand		50	70	30	60	

An example with a dummy source (30)

Table 8.12 Parameter table for Metro water district

	Cost (Tens of Millions of Dollars) per unit distributed					Supply
	Destination					
Source	Berdoo 1	Berdoo (extra) 2	Los Devils 3	San Go 4	HollyGlass 5	
Colombo river 1	16	16	13	22	17	50
Sacron River 2	14	14	13	19	15	60
Calories River 3	19	19	20	23	M	50
Dummy 4(D)	M	0	M	0	0	50
Demand	30	20	70	30	60	

TRANSPORTATION PROBLEMS (31)

- Initial Solution from Northwest Corner Rule

Source	Destination 1	2	3	4	5	Supply
1	16	16	13	22	17	50
2	14	14	13	19	15	60
3	19	19	20	23	M	50
4 (D)	M	0	M	0	0	50
Demand	30	20	70	30	60	

Solution:
Total Cost (Z) =

BV:

Non-BV: Other $X_{ij} = 0$

TRANSPORTATION PROBLEMS (32)

- Initial Solution from Least Cost Method

Source	Destination 1	2	3	4	5	Supply
1	16	16	13	22	17	50
2	14	14	13	19	15	60
3	19	19	20	23	M	50
4 (D)	M	0	M	0	0	50
Demand	30	20	70	30	60	

Solution:

BV:

Non-BV: Other $X_{ij} = 0$

Transportation Problems (33)

- Initial Solution from VAM (1)

Source	Destination 1	2	3	4	5	Supply
1	16	16	13	22	17	50
2	14	14	13	19	15	60
3	19	19	20	23	M	50
4 (D)	M	0	M	0	0	50
Demand	30	20	70	30	60	

Penalty					

Penalty					

Transportation Problems (34)

- Initial Solution from VAM (2)

Source	Destination 1	2	3	4	5	Supply
1	16	16	13	22	17	50
2	14	14	13	19	15	60
3	19	19	20	23	M	50
4 (D)	M	0	M	0	0	50
Demand	30	20	70	30	60	

Solution:

BV:

Non-BV: Other $X_{ij} = 0$

Transportation Problems (35)

- Check Optimality from VAM Solution

U_i	Source	Destination 1	2	3	4	5	Supply
	1	16	16	13 / **50**	22	17	50
	2	14 / **0**	14	13 / **20**	19	15 / **40**	60
	3	19 / **30**	19 / **20**	20	23	M	50
	4 (D)	M	0	M	0 / **30**	0 / **20**	50
	Demand	30	20	70	30	60	
	V_j						

Transportation Problems (36)

- Optimal Solution:
  - Total Cost (Z) = 2460
  - BV: X_{13}= 50, X_{21}= 0, X_{23}= 20, X_{25}= 40, X_{31}= 30, X_{32}= 20, X_{D4}= 30 and X_{D5}= 20
  - Non-BV: Other X_{ij} = 0

Transportation Problems (37)

- So, from our problem: BV: X_{13}= 50, X_{21}= 0, X_{23}= 20, X_{25}= 40, X_{31}= 30, X_{32}= 20, X_{D4}= 30 and X_{D5}= 20

TABLE 8.10 Water resources data for Metro Water District

	Cost (Tens of Dollars) per Acre Foot				
	Berdoo	**Los Devils**	**San Go**	**Hollyglass**	**Supply**
Colombo River	16	13	22	17	50
Sacron River	14	13	19	15	60
Calorie River	19	20	23	—	50
Minimum needed	30	70	0	10	(in units of 1 million acre feet)
Requested	50	70	30	∞	

- Berdoo (1,2) get 50 from Calorie River.
- Los Devils (3) get 70 from Colombo and Sacron River.
- There is no supply for San Go (4). (Min needed = 0)
- Hollyglass (5)gets 40 from Sacron River. (Min needed =10)

Q&A

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